

## LETTERS TO THE EDITOR.

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### The Probable Error of the Amplitudes in a Fourier Series obtained from a Given Set of Observations.

MR. DINES'S question (NATURE, August 12, p. 644) as to the degree of significance to be attached to the amplitudes of waves of given period found in short series of observations and the likelihood of their permanence as the series is extended brings home again the reflection that whilst probable errors are easily found where the experience is large and they are least wanted, their determination from a limited experience where they would be most useful is largely hypothetical.

One of the principal sources of error springs from the process of selection. For example, we may find by counting that twelve persons out of twenty in an omnibus, picking up people at random, are males; but to predict from this the probable constitutions in like respect of other omnibuses, or to say that the percentage of males in London has been determined as sixty, with a probable error of 7.5, is clearly a misuse of statistical theory. The result is fallacious, because of the false premises. Omnibuses do not pick up people at random, or even travelling people at random. Persons go about in small groups, and there are larger groups going to the shopping centres and going citywards, and still larger groups travelling at the extreme hours and travelling at the mid hours of the day, and so on. The constitutions of the groups are different, and the probable error of the percentage is much in excess of that calculated on the basis of random sampling, this being due to a process which, applied to phenomena in general, may be called a tendency to cluster.

In all cases where a space or time content is taken as the sample it is necessary, owing to unknown clustering, to repeat the samples many times over, before a just estimate of the statistical constants and of their probable errors can be obtained.

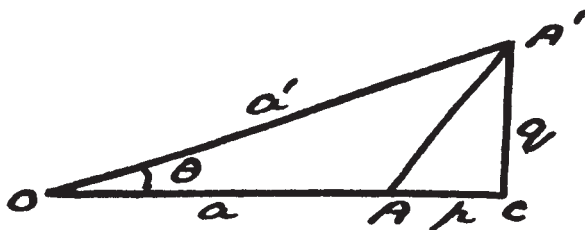
Applied to meteorological and such-like phenomena, in which some value fluctuates in time, the successive observations are seldom at such a distance as to be haphazard, and the clustering, in this instance, is of the nature of waves similar to the waves which are the subject of observation. In these circumstances it does not seem possible, from observations of a single period, to make any estimate of the significance of an observed amplitude. This can only be done by repeating the period and noting the fluctuations in phase and amplitude found in such repetitions. If observations for many periods are to hand, then it is only following a well-understood practice to divide the material up into groups and calculate the constants of each group. From the fluctuations found in the several groups an empirical gauge of error may be constructed, upon which may be based a measure of error, suitable for application to the constant found for the whole.

If the phenomenon were one fully observed, and a frequency distribution of wave period and amplitude had been calculated by one of the several methods that have been proposed, then it would not be difficult to obtain the probability of error of any sustained wave found in the observations during a limited period of time. But such a frequency distribution is seldom a subject of research, although it is the first

step in a description of the phenomenon, and one that cannot fail, by giving the salient periods, to mark out regions of investigation relative to the causes of undulation.

The more limited statement of the problem, as enunciated by Mr. Dines, renders it amenable to algebraical expression in the following manner.

A variable quantity has a periodic wave of amplitude  $a$ , and is subject, in addition, to casual fluctuations the mean of which is zero, and standard deviation  $\sigma$ . It is easily shown that if  $p$  and  $q$  are the amplitudes of the waves, phasal with, and in quadrature with, the above wave, calculated from the casual variations alone, the mean values of  $p$  and  $q$  are zero, and their standard deviations are  $\sigma\sqrt{2}/\sqrt{n}$ . Supposing that the distributions of  $p$  and  $q$  are normal, what will be the errors of the calculated amplitudes, and phase angles, due to these fluctuations?



Draw  $OA=a$ ,  $AC=p$ ,  $CA'$  at right angles  $=q$ .

Then, clearly,  $a' = OA'$  is the calculated amplitude due to  $p$  and  $q$ , superimposed upon  $a$ , and  $\theta = AOA'$  is the phase displacement.

Now  $A'$  has varying positions, due to the variations of  $p$  and  $q$ , and its frequency upon the element of area,  $dp dq$ , about  $A'$ , is that of a normal distribution, viz. :—

$$\frac{1}{\sqrt{(2\pi)s}} e^{-\frac{1}{2}p^2/s^2} dp \cdot \frac{1}{\sqrt{(2\pi)s}} e^{-\frac{1}{2}q^2/s^2} dq$$

$$= \frac{1}{2\pi s^2} e^{-\frac{1}{2}(p^2+q^2)/s^2} dp dq$$

where  $s$  is written for  $\sigma\sqrt{2}/\sqrt{n}$ , the s.d. of  $p$  or  $q$ .

Putting  $p^2 + q^2 = AA'^2 = a'^2 - 2a'a \cos \theta + a^2$ , and  $dp dq = a' da' d\theta$ , the variables are changed from  $p, q$  to  $a', \theta$ , and the frequency distribution of  $a', \theta$  is therefore

$$\int_0^{2\pi} \int_0^\infty \frac{1}{2\pi s^2} e^{-\frac{1}{2}(a'^2 - 2a'a \cos \theta + a^2)/s^2} a' da' d\theta.$$

When this is integrated with respect to  $\theta$  the result will be the frequency distribution of amplitudes  $a'$ . It appears to be a problem of the nature of random migration, and the solution of the above integral will probably be found (I am unable to consult my reference) in Prof. Karl Pearson's memoir upon this subject.

If there is no initial wave, so that the calculated amplitudes are all due to casual fluctuations, then  $a=0$  and the frequency distribution of  $a'$  becomes :—

$$\int_0^\infty \frac{1}{s^2} e^{-\frac{1}{2}a'^2/s^2} a' da'.$$

This is a well-known distribution, being, for instance, the distribution of arithmetical velocity of wind when the NS and EW components vary independently, in normal manner, with s.d. equal to  $s$ .

The mean is  $\sqrt{(\frac{1}{2}\pi)} \cdot s = 1.253s$ , the standard deviation is  $\sqrt{(2 - \frac{1}{2}\pi)} \cdot s = 0.655s$ , and the number per cent. exceeding  $xs$  is  $100 \times e^{-\frac{1}{2}x^2}$ .

Applying these formulæ to Mr. Dines's experimental data, the comparison of experiment with theory comes out as follows :—

Values of  $a' = \sqrt{(p^2 + q^2)}$ . Mean  $p = \text{mean } q = 0$ .  
*S.d.* of  $p = \text{s.d. of } q = 10$ . 500 pairs taken.

|                                | Experiment | Theory           |
|--------------------------------|------------|------------------|
| Mean $a'$ ... ..               | 12.5       | $12.53 \pm 0.20$ |
| Standard deviation of $a'$ ... | 6.5        | $6.55 \pm 0.14$  |
| Per cent. exceeding 25 ...     | 4.0        | $4.39 \pm 0.62$  |

the agreement being very close in all three values.

Manchester.

H. E. SOPER.

#### Antarctic Fossil Plants.

It should have been stated that Prof. Seward's memoir (reviewed in *NATURE* for August 26) is the first dealing with the *geological* results of Capt. Scott's Expedition. Two numbers of the Zoological series had previously appeared—No. 1 of vol. i. on June 27, and No. 1 of vol. ii. on July 25, 1914.

D. H. S.

#### HENRY GWYN JEFFREYS MOSELEY.

SCIENTIFIC men of this country have viewed with mingled feelings of pride and apprehension the enlistment in the new armies of so many of our most promising young men of science—with pride for their ready and ungrudging response to their country's call, and with apprehension of irreparable losses to science. These forebodings have been only too promptly realised by the death in action at the Dardanelles, on August 10, of Henry Gwyn Jeffreys Moseley, 2nd Lieut. in the Royal Engineers, at the age of twenty-seven. A son of the distinguished zoologist, the late Prof. H. N. Moseley, of Oxford, he was educated at Eton, entering as a scholar, and passed to Trinity College, Oxford, where he gained a Millard Scholarship. He obtained a First Class in Mathematical Moderations, and Honours in Natural Science.

Moseley early showed marked originality and an enthusiastic interest in science. A year before his graduation he had decided to undertake original work in physics, and visited Manchester to discuss the matter with me. After graduation, he was appointed lecturer and demonstrator in the physics department of the University of Manchester, and immediately devoted all his spare time to investigation. After two years he resigned his lectureship in order to devote his energies entirely to research, and was awarded the John Harling Fellowship. During the past year he went to Oxford to live with his mother, and to continue his experiments in the laboratory of Prof. Townsend. He went out to Australia with the British Association, took an active part in the discussion on the "Structure of the Atom" at Melbourne, and gave an interesting account of his recent work on the X-ray spectra of the rare earths, in Sydney. On the outbreak of war he put aside all thought of continuing the investigations in which he was so vitally interested, and returned at once to England to offer his services to his country, and was granted a Commission in the Royal Engineers. He was later made signalling officer to the 38th Brigade of the First Army, and left for the Dardanelles on June 13. He took part in the severe fighting at the new landing on

August 6 and 8, and was instantaneously killed on the 10th by a bullet through the head in the act of telephoning an order to his division at a moment when the Turks were attacking on the flank only 200 yards away.

Moseley was one of those rare examples of a man who was a born investigator. He rapidly acquired the technique of experiment and soon gained a remarkably wide and accurate knowledge of modern physics. His undoubted originality and marked capacity as an investigator were very soon ungrudgingly recognised by his co-workers in the laboratory, while his cheerfulness and willingness to help in all possible ways endeared him to all his colleagues. His first research, published in the Proceedings of the Royal Society, consisted in the determination of the average number of beta particles emitted during the transformation of an atom of radium B and radium C—a difficult and important piece of work. It then occurred to him to determine the potential to which radium could be charged in a high vacuum by the escape of its own beta particles. He was able to achieve such a high stage of exhaustion—and this before the advent of the molecular pump—that a small quantity of radioactive matter retained itself at a potential of more than 100,000 volts for several weeks. He devised an ingenious method for detecting the possible presence of very short-lived radioactive substances, and in conjunction with Fajans utilised the method to determine the period of transformation of a newly-discovered product in actinium, which was found to be half transformed in  $1/500$  of a second.

Moseley's interest was greatly aroused by the discovery of Laue of the diffraction of X-rays in their passage through crystals, and in conjunction with Mr. Charles Darwin he immediately started an investigation to examine the quantity and quality of the X-radiation scattered from crystals at different angles. Prof. Bragg, who was working simultaneously at Leeds on the same problem, observed the presence of definite maxima in the scattered radiation corresponding to definite lines in the X-ray spectrum. This result was confirmed and extended by Moseley and Darwin, and they mapped out accurately for the first time the spectrum of the characteristic X-radiation from an X-ray tube with a platinum antikathode. These pioneer investigations in Leeds and Manchester were of fundamental importance, for they laid the foundation of the new science of X-ray spectroscopy, which is now in the process of rapid development.

Moseley next decided to examine the X-ray spectra of a large number of different elements with the definite object of testing whether the spectrum was connected in a simple way with the atomic number of the element when arranged in increasing order of atomic weight. Suggestions had been previously made that the charge on the nucleus of an atom, which defines its chemical and physical properties, was possibly equal to the atomic number. For this purpose he developed the photographic method for accurate measure-